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Observatory, Atlas Photographique de la Lune, fasc. 10, presented by the Observatory; R. Patxot Jubert, Observaciones de San Feliu de Guixols, presented by the author; B. O. Peirce, Theory of the Newtonian Potential Function, presented by Mr. Frank Robbins; A. Stanley Williams, Zenographical Fragments, part 2, presented by the author.

Astrographic Chart; 36 charts, presented by the Royal Observatory, Greenwich; 20 charts, presented by the San Fernando Observatory, 14 by the Tacubaya Observatory, and 22 by the French Minister of Public Instruction, from the Bordeaux and Toulouse Observatories.

On Correlation and the Characters of Variable Stars: in reply to Professor Karl Pearson. By H. C. Plummer, M.A.

1. The paper* in which Professor Pearson has discussed some of the statistical relationships to which different classes of variable stars may be subject has natural claims to our respectful attention. Much work in modern astronomy, whether expressly inspired by the evolutional point of view or not, is of the nature of classification. Professor Pearson has done so much to provide the necessary theoretical basis for studying the correlation of classified characters that we should be grateful to him for dealing with some of the problems connected with variable stars which are of this kind, and thereby drawing more general attention to a branch of astrophysics which is of the greatest interest and importance.

At the same time it seems to me that some of Professor Pearson's results stand in need of serious qualification, being open to a very simple line of criticism (see §§ 3, 4), which he has apparently overlooked. If this be conceded, these results must lose much of their value, and no very great confidence can be placed in certain ulterior speculations which have been based on them. This may be thought a harsh judgment. But if it can be shown that in this instance Professor Pearson, possibly misled by a certain want of familiarity with the subject of variable stars, has made a not altogether judicious use of his own machinery, it will surely be better to vindicate the general methods, even at the expense, if need be, of the particular results.

Perhaps an example will serve to explain, if it does not justify, the feeling that such a critical attitude is necessary. In the case of long-period variables, Professor Pearson finds a correlation between the light-range and maximum light of seventy-nine stars which is little short of unity (·9996). Now if this means anything at all, it should mean that the light-range is almost rigorously a linear function of the maximum light. But if this be so, a similar relation ought to hold between the maximum light and the minimum light. Now this is not so. Hence it is impossible to

* *M.N.*, lxix. p. 128 (1908 Dec.).

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draw any conclusion of value from the figure found for the correlation, the magnitude of which, it may be suggested, is fully accounted for by the simple fact that the minimum light is very small in comparison with the maximum light for this class of stars. The biologist may perforce be content with the knowledge that a relationship exists: the astronomer is not accustomed to rest content until he knows something about the nature of the relationship. He will certainly not be attracted by modern methods in statistics if they lead to results from which no useful deduction can be made, as they appear to do in the example quoted. It is therefore of great importance that we should scrutinise carefully the application of a form of statistical calculus which itself is no doubt capable of doing useful service.

2. One point of a general character must be considered before we come to any question of statistics. Professor Pearson insists, not for the first time, that it is "absolutely needful to use the true physical variable amount of luminosity, and not the merely physiological variable magnitude." By what canon, it may be asked, is he able, as he seems to be, to distinguish *a priori* between these two classes of variables? One is purely a mathematical function of the other, and it is readily conceivable that the result of a statistical discussion would be to show what particular function of a variable would be most appropriate for use in a particular problem; that, in fact, this might be the most useful outcome of such a discussion. But in Professor Pearson's paper the discrimination appears as a matter of *a priori* judgment, and not at all as a tentative suggestion to be verified by experience. This view of his attitude may be mistaken, but it seems probable that there is a real preconception on his part with regard to this point, and that it has prejudiced both his treatment of the statistics and his interpretation of the results.

However this may be, the phrase quoted above seems to have been expressed a little unfortunately. The astronomer is perfectly ready to admit the detrimental effect of "physiological" influences. But there are two reasons why the use of the word is not very happy in this particular case. The determinations of magnitude can be made photographically, and not visually at all; indeed, they have been so made by Professor S. I. Bailey, whose results for the ω Centauri variables are employed by Professor Pearson. Again, it is no question of the errors in the determinations which may arise from this—or any other—source. For such errors would be reproduced in the deduced luminosities, and the latter are accepted without the slightest suggestion that they are in any way inaccurate.

On the other hand, the expression "true physical variable" may easily cover a false assumption. In dealing with variable stars, it must be borne in mind that in no case is there complete certainty that we possess the true explanation of the light-fluctuation. The use of a term like "obscuration" begs the whole question. Even in the case of the Algol variables, the eclipse theory can claim no more than a high degree of probability—a very

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high degree, no doubt, but one falling distinctly short of certainty. If we consider the light-curves for the Algol and for the antalgol types, we might expect to find some analogous features in the causes underlying the two classes; by accepting the eclipse theory for the former, we apparently exclude such a possibility. Of course we know that Algol is a spectroscopic binary, but as much may probably be said of every short-period variable. This argument has therefore little weight, especially when we notice* that the reconciliation of photometric and spectrographic observations of Algol is far from being the certain and simple matter we could desire. But if there is room for so much as the slightest suspicion in the case of Algol itself, what is to be said of the other types discussed by Professor Pearson?

When the physical cause underlying the variability is entirely unknown, both magnitude and luminosity are merely measures of some physical change. To discover which is the more appropriate measure is a matter of experiment, and the experiment must be repeated with every fresh line of discussion. On the commonly accepted theory of Algol variables, Professor Pearson is no doubt justified in choosing the luminosity as the more appropriate variable. But were the type of variation that which would be caused by the varying thickness of a stellar atmosphere producing uniform and general absorption of the star's light, it is easily conceivable that the star's magnitude would be the more appropriate variable, as directly connected with the thickness of the atmospheric envelope. And this is merely a random suggestion of one possibility.

3. We can now turn to the point in the theory of statistics which Professor Pearson has apparently overlooked. In this we are taking advantage of a clue which he has himself given us, for it is he who has drawn our attention to the importance of spurious correlation. We have, writing $\sigma(x)$ for the standard deviation of x, and $r(x, y)$ for the coefficient of correlation between x and y,

$$\begin{aligned}\sigma^2(x-y) &= \sigma^2(x)+\sigma^2(y)-2r(x, y)\sigma(x)\sigma(y)\\ \sigma^2(x) &= \sigma^2(y)+\sigma^2(x-y)+2r(y, x-y)\sigma(y)\sigma(x-y)\\ \sigma^2(y) &= \sigma^2(x)+\sigma^2(x-y)-2r(x, x-y)\sigma(x)\sigma(x-y)\end{aligned}$$

The form of these equations suggests a simple geometrical interpretation. Consider a triangle A B C whose sides are A B $=\sigma(x)$, A C $=\sigma(y)$, and B C $=\sigma(x-y)$. It follows that

$$r(x, y)=\cos \mathrm{A}, \quad r(x, x-y)=\cos \mathrm{B}, \quad r(y, x-y)= -\cos \mathrm{C}.$$

Now one angle of a triangle at least must be less than 60°. Hence one at least of the three coefficients of correlation must be greater than $\cos 60° = 0{\cdot}5$, and this number is supposed to indicate appreciable correlation when the quantities are independent. But if one of them, say $r(x, y)$, is small, which will be the case if x

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* *Cf.* R. H. Curtiss, *Astrophys. Jour.*, xxviii. p. 150; Schlesinger and Curtiss, *Publ. Allegheny Obs.*, vol. i., No. 5.

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and y be in no way related, then A will be nearly 90°, and either B or C must be 45° or less, so that one at least of the other correlation coefficients, say $r(x, x-y)$, will exceed numerically $1/\sqrt{2}$, or 0·707. This explains how quite a large coefficient of correlation can be obtained which is wholly spurious.

But this is not all. It is evident that

$$\sin B < \sigma(y)/\sigma(x),$$

and therefore, if $\sigma(x) > \sigma(y)$,

$$r(x, x-y) = \cos B > \sqrt{\{\sigma^2(x) - \sigma^2(y)\}}/\sigma(x).$$

Let B_0 be the greatest value of B consistent with this inequality. Then we see that if x and y represent any two series of numbers whatever, and a perverse imp were to tamper with the order of the two series without changing the numbers themselves in any way, so as to make the coefficient of correlation between x and $x-y$ the smallest possible, he could not by any possibility make it less than $\cos B_0$; and unaided nature cannot be expected to reduce it even to this limit.

4. Let us compare, then, the values of $\cos B_0$ in the several cases with the correlations obtained by Professor Pearson.

Cluster Variables (91 Stars).

$$\sigma(L_2) = {\cdot}0137, \quad \sigma(L_1) = {\cdot}0051, \quad \sigma(L_2 - L_1) = {\cdot}0134$$

$$\left.\begin{aligned} B_0 = 21^\circ\ 51', \quad \cos B_0 &= {\cdot}9281 \\ r(L_2 - L_1, L_2) &= {\cdot}9283 \end{aligned}\right\}$$

Algol Type (57 Stars).

$$\sigma(L_2) = 168{\cdot}16, \quad \sigma(L_1) = 60{\cdot}08, \quad \sigma(L_2 - L_1) = 109{\cdot}33$$

$$\left.\begin{aligned} B_0 = 20^\circ\ 56', \quad \cos B_0 &= {\cdot}9340 \\ r(L_2 - L_1, L_2) &= {\cdot}9959 \end{aligned}\right\}$$

Algol Type (54 Stars).

$$\sigma(L_2) = 15{\cdot}79, \quad \sigma(L_1) = 6{\cdot}15, \quad \sigma(L_2 - L_1) = 10{\cdot}14$$

$$\left.\begin{aligned} B_0 = 22^\circ\ 55', \quad \cos B_0 &= {\cdot}9211 \\ r(L_2 - L_1, L_2) &= {\cdot}9837 \end{aligned}\right\}$$

Long-Period Variables (87 Stars).

$$\sigma(L_2) = 15{\cdot}756, \quad \sigma(L_1) = 3{\cdot}446, \quad \sigma(L_2 - L_1) = 14{\cdot}870$$

$$\left.\begin{aligned} B_0 = 12^\circ\ 38', \quad \cos B_0 &= {\cdot}9758 \\ r(L_2 - L_1, L_2) &= {\cdot}9763 \end{aligned}\right\}$$

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Long-Period Variables (79 Stars).

$$\sigma(L_2) = 15{\cdot}582, \quad \sigma(L_1) = {\cdot}515, \quad \sigma(L_2 - L_1) = 15{\cdot}357$$

$$\left.\begin{array}{r} B_0 = 1^\circ\ 54', \quad \cos B_0 = {\cdot}9995 \\ r(L_2 - L_1, L_2) = {\cdot}9996 \end{array}\right\}$$

5. A simple inspection of these figures suggests that Professor Pearson has placed a false interpretation on his results. The high correlations obtained do not correspond with any close physical relationship between maximum light and light-range, but are merely expressions of the fact that the variability of the maximum light is very much greater than the variability of the minimum light. This fact itself seems significant, and, taken by itself, would suggest that the efficient cause behind the variation of a star's light is in general of an additive rather than of a subtractive or "obscuring" kind. No undue stress need be laid on this remark, but it will be remembered that Professor Turner,* in attempting to bring the Sun into line with long-period variables, was led to identify Sun-spot maximum with star-light maximum,—not star-light minimum, as Professor Pearson seems by his language to assume as a matter of course. However this may be, it is evident that spurious correlation has played so large a part that, with the possible exception of the Algol variables, no room is left for the influence of any physical correlation between maximum light and light-range. Even in the case of the Algol variables, the figures not only prove that the correlation need not be particularly large, but even suggest that it is surprisingly small. At any rate, if sensible correlation exists, the method followed tends rather to conceal it than to bring it to light. It is unnecessary to follow Professor Pearson in the inferences which he has based on the large values found for $r(L_2 - L_1, L_2)$.

6. So far as the relationship between range and maximum brightness is concerned, it does not appear that the observed magnitudes furnish an index which is necessarily inferior to that supplied by the deduced luminosities. A trial has been made in the case of the short-period variables of the ω Centauri cluster, with the following result:—

Cluster Variables (91 stars).

$$\sigma(m_2) = 0{\cdot}234, \quad \sigma(m_1) = 0{\cdot}197, \quad \sigma(m_2 - m_1) = 0{\cdot}293$$

$$\left.\begin{array}{r} B_0 = 57^\circ\ 21', \quad \cos B_0 = {\cdot}5395 \\ r(m_2, m_2 - m_1) = {\cdot}7540 \end{array}\right\}$$

The apparent correlation is much less than that which results from the luminosities, but the certainly spurious element is also much less, so that room is left for the effect of a small but possibly real correlation. The example tends to show that star-magnitudes

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* *M.N.*, vol. lxiv. p. 550.

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are not so obviously unworthy of direct statistical treatment as Professor Pearson has implied.

7. If we except the Algol type stars, the figures for the correlation between period and light-fluctuation seem to lead to a similar conclusion. For the results obtained by Professor Pearson and Professor Turner (p. 138) give—

Type.	No. of Stars.	r(P, F).	$r(\mathrm{P}, m_1 - m_2)$.
Cluster . .	91	·457	42
Long-period . .	87	·049.	
,, . .	79	·201	

For the cluster variables the results are practically the same; for the long-period variables the correlation between period and fluctuation is too small to be of any serious importance. The correlation between period and magnitude range has not been calculated for the long-period variables. But it is unnecessary to do this in order to see that we have no evidence that there is any advantage in using the light-fluctuation rather than the magnitude range in dealing with either of these two classes of variable stars.

8. There remains for brief consideration the apparent exception in the case of the Algol stars. Without excluding any of the 57 stars employed, Professor Pearson finds that

$$r(\mathrm{P}, \mathrm{F}) = {\cdot}4109, \quad \eta(\mathrm{P}, \mathrm{F}) = {\cdot}57, \quad r(\mathrm{P}, m_1 - m_2) = {\cdot}26$$

The regression line is curved, and the correlation coefficient is not a satisfactory measure, therefore, of the relationship between P and F, nor, presumably, of the relationship between P and $m_1 - m_2$. The nature of the curved regression line is not very clearly indicated by the data, only 15 stars having a period of more than 4 days, and it is a little difficult to judge how far the results are affected by the inadequacy of the material, and by the natural selection which depends upon the circumstances favourable to the detection of variability. The results of S. Beljawsky, quoted by Professor Pearson, refer to a different class of stars, with a quite different range of periods, and have no bearing whatever on the present question. The point of interest suggested by the present discussion of the Algol stars is, in view of its bearing on the eclipse theory, that apparently the period is *positively* correlated with light-fluctuation. The "ready explanation" which Professor Pearson suggests without formulating, and then criticises, has not occurred to me, and it does not seem to require two or three pages of mathematical reasoning to see that the increase of light-fluctuation with period is altogether surprising, if it be real. For if we take a given system and, *ceteris paribus*, increase the period and therewith the mean distance, the result will certainly be to diminish the fluctuation, unless we observe from a point exactly in the plane of the orbit. Professor Pearson's suggestion, to meet the difficulty, is that a large orbit implies a large eclipsing body. The idea is an interesting one, and it is not impossible that some light might

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be thrown on the question by a careful study of a large number of light-curves. Perhaps an alternative process of physical development may be suggested. Consider a system with a relatively small orbit and short period. The components may be supposed at first to be of approximately equal luminosity. With the lapse of time, the operation of tidal action will gradually increase the period and the distance between the bodies. Meanwhile, unless the components are of equal mass and similar constitution, a difference of luminosity may be expected to develop between the components, and the effect of increasing contrast may suffice to counterbalance the effect of increasing distance on the light-fluctuation. If such an explanation were valid, it is again possible that confirmation might be supplied by a study of the elements of a large number of stars, but at present we have scarcely enough material to settle such a question.

9. As already stated, the facts by which such conjectures are prompted are much obscured by the uncertainties to which random and natural selection may give rise. Moreover they have been interpreted in the light of the eclipse theory, and it is quite conceivable that some stars have been included to which this theory does not apply. Even a superficial inspection of the cases which are commonly admitted to a list of Algol variables is enough to suggest that a distinct advantage would be gained if a clear definition of what constitutes an Algol variable could be adopted. Such a definition must be framed with the object of separating the true Algol type from the β Lyræ type. This is not an easy thing to do perhaps, and it will probably be necessary to have an intermediate class in which the doubtful cases can be kept either temporarily or permanently. But whatever may be the difficulties in the way of a precise classification, it is hard to see, for example, why such a star as V Puppis should be associated with β Persei and separated from β Lyræ. The most promising line of attack is by the properly coordinated application of both the spectrograph and the photometer to members of both types. Coordination has hitherto been lacking, so that it has happened that when a spectrographic orbit has been worked out, the corresponding photometric data have either been entirely absent or of an accuracy not sufficient to inspire complete confidence. It is true that the field of such work is restricted and hampered by the faintness of the stars and the early type of spectra characteristic of the Algol class. But I may perhaps express the belief that more will be done by the most careful study of a few typical examples than by the mere accumulation of a mass of uncoordinated material and the indiscriminate volley-firing, so to speak, of a statistical method.

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On the Testing of Paraboloidal Mirrors.

By Rev. C. D. P. Davies, M.A. (Plates 14, 15.)

1. *Introduction.*

In view of the fact that the Society's publications contain nowhere any connected account of the testing of paraboloidal mirrors, it seems desirable to give the theory and its practical application.

2. *References.*

The following references may be found useful. They are given in chronological order.

(1) Léon Foucault, in vol. v. of *Annales de l'Observatoire de Paris* (*Mémoires*), 1859, published an account of his testing mirrors at their centre of curvature. This paper forms the basis of the modern method of testing.

(2) Henry Draper, M.D. (1864), gives a fragmentary description of portions of the subject in vol. xiv. of *The Smithsonian Contributions to Knowledge.* This is reprinted in vol. xxxiv.; see specially section "*d*," pp. 13–19.

(3) H. A. Wassell gives the fullest and best description hitherto published in the form of letters to the *English Mechanic.* See Nos. 887, 890, 894, 899, 902, 907, 911, 962, 1078, 1089, and 1129, ranging from April 1882 to November 1886.

(4) Dr. A. A. Common, F.R.S., makes some remarks on the matter in *R.A.S. Memoirs*, vol. l. pp. 149–158.

(5) A. Woolsey Blacklock, M.D., gives a series of five excellent and reliable letters in Nos. 1579, '81, '83, '85, and '88 of the *English Mechanic* (1895).

(6) Professor G. W. Ritchey writes on the subject in vol. xxxiv. of *The Smithsonian Contributions to Knowledge*; see especially pp. 30–38 (1904).

3. *Principle of the test.*

The essential feature of the operation of testing a parabolic mirror consists in the fact that it is conducted at the centre of curvature, and not, as in the days before Foucault, at its principal focus.

The mirror is illuminated by a point of light placed as close as possible to the axial line of the mirror, and at a distance from the mirror equal to its radius of curvature. Under these circumstances an image of the pin-hole is formed at the same distance from the mirror as the pin-hole, and as far on the one side of the axial line as the pin-hole is on the other side of it. The pinhole and its image should always be brought as near to each other as is practically possible. On placing the eye at the image of the pin-hole the face of the mirror is seen illuminated, and can be scrutinised in detail.

4. *Apparatus.*

For effecting this scrutiny three pieces of apparatus are required. These are (1) the lamp, (2) the measuring machine, and (3) the support for the mirror.